

## DYNAMICAL LOW-MASS FERMION GENERATION IN RANDALL-SUNDRUM BACKGROUND \*

T. INAGAKI

*Information Media Center, Hiroshima University,  
Higashi-Hiroshima, 739-8521, Japan  
E-mail: inagaki@hiroshima-u.ac.jp*

It is investigated that a dynamical mechanism to generate a low mass fermion in Randall-Sundrum (RS) background. We consider a five-dimensional four-fermion interaction model with two kinds of bulk fermion fields and take all the mass scale in the five-dimensional spacetime at the Planck scale. Evaluating the effective potential of the induced four-dimensional model, I calculate the dynamically generated fermion mass. It is shown that dynamical symmetry breaking takes place and one of the fermion mass is generated at the electroweak scale in four dimensions.

### 1. Introduction

To construct a unified theory of the electroweak interaction, strong interaction and gravity it is important to make investigation on the gauge hierarchy problem, how the electroweak scale is realized in the theory at the Planck scale. As in a large extra-dimension model it is possible to solve the gauge hierarchy problem to consider a four-dimensional brane embedded in a higher-dimensional spacetime.<sup>1,2</sup> Randall and Sundrum considered a higher-dimensional curved spacetime with negative curvature and found a beautiful solution of the hierarchy problem by using the exponential factor in the metric.<sup>3</sup> Here we launched a plan to study a dynamical mechanism to realize the electroweak scale from the Planck scale physics in a model of the brane world proposed by Randall and Sundrum.<sup>4,5,6,7</sup>

At the beginning, it is considered that only the graviton can propagate in the extra-dimension and all the standard model particles are localized on the four-dimensional brane. However, there is a possibility that some of the standard model particles also propagate in the extra-dimension.<sup>8</sup> In

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Fig. 1 we illustrate an image of a four-dimensional brane embedded in a five-dimensional bulk. The bulk fields are the fields which can propagate in the bulk. The KK excitation modes of the bulk fields appear on the brane and the modes may affect some of low energy phenomena.

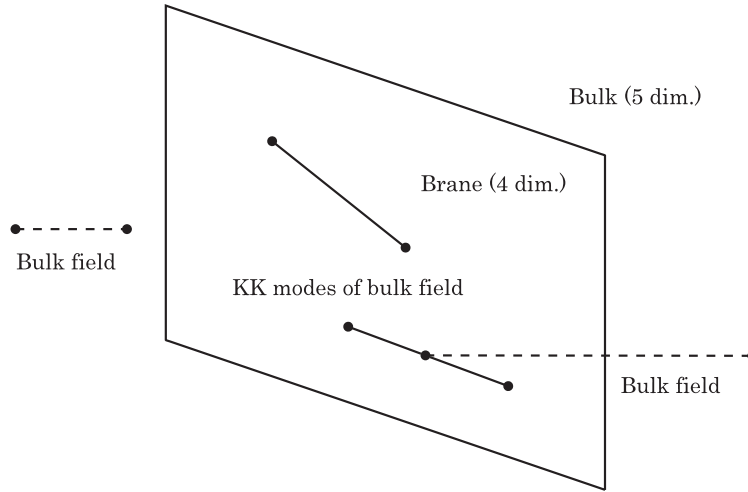


Figure 1. Image of the brane world.

One of the interesting phenomena is found in a spontaneous electroweak symmetry breaking. The electroweak symmetry can be dynamically broken down due to the fermion and anti-fermion condensation. Many works have been done to see the contribution of the KK modes to dynamical symmetry breaking in models with large extra dimensions.<sup>9,10,11,12,13,14,15</sup> Here a theory with bulk fermions is considered in the RS background. We assume the existence of two types of bulk fermion fields which can propagate in a five-dimensional bulk. To construct a model where the fermion field naturally develops the electroweak mass scale, a four-fermion interaction is introduced between these bulk fermions. As is known, the four-fermion interaction model is a simple model of dynamical symmetry breaking. It is expected that a negative curvature enhances symmetry breaking.<sup>16,17,18,19</sup> Evaluating the induced four-dimensional effective potential, we calculate the mass scale of the fermion in four dimensions. Since we are interested in the bulk standard model particles, the KK excitations of graviton are assumed to have no serious effect on the fermion mass and ignore them.

## 2. Four-Fermion Interaction Model in Randall-Sundrum Background

Here we briefly review the Randall-Sundrum idea<sup>3</sup> and introduce a four-fermion interaction between bulk fermions.

### 2.1. *Randall-Sundrum Background*

The RS background is a five-dimensional spacetime whose fifth dimension is compactified on an orbifold with  $S^1/Z_2$  symmetry and two Minkowski branes exist at the orbifold fixed points,  $\theta = 0$  and  $\pi$ , see Fig. 2. The

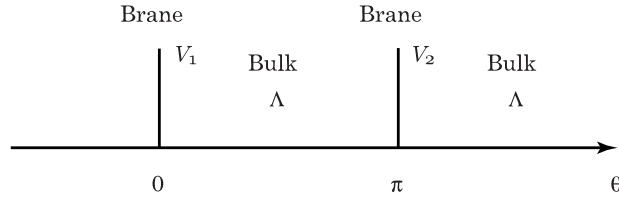


Figure 2. Randall-Sundrum spacetime

background spacetime is a static solution of the Einstein equation if the cosmological constant in the bulk,  $\Lambda$ , and that on the brane,  $V_1, V_2$ , satisfy the relationship,

$$\Lambda = -V_1 = V_2. \quad (1)$$

The spacetime described by the metric,

$$g^{\mu\nu} = e^{-2kr|\theta|} \eta_{\mu\nu} dx^\mu dx^\nu + r^2 d\theta^2. \quad (2)$$

It is a maximally symmetric spacetime with a constant negative curvature, i.e., five-dimensional anti de-Sitter spacetime.

The warp factor  $e^{-2kr|\theta|}$  in Eq.(2) plays an important role to solve the hierarchy problem. The effective Planck scale, mass scale for gravity on the brane, is given by

$$M_{pl}^2 \sim \frac{M^3}{k} (1 - e^{-2kr\pi}) \sim \frac{M^3}{k}, \quad (3)$$

where  $M$  is the fundamental scale in the bulk and  $k$  is the curvature. On the other hand, the mass scale for  $M_{phys}$  on the  $\theta = \pi$  brane is suppressed by the warp factor.

$$M_{phys} = M e^{-kr\pi}. \quad (4)$$

For  $k \sim 11$ , the electroweak mass scale,  $M_{EW}$ , can be realized from only the Planck scale,  $M_{pl}$ , without introducing some large number.

$$\begin{cases} M \sim k \sim O(M_{pl}), \\ M_{phys} \sim O(M_{EW}). \end{cases} \quad (5)$$

This is the most important mechanism of the RS model. We want to realize this mechanism dynamically and construct a model where the fermion mass is generated at the electroweak scale.

## 2.2. Bulk Four-Fermion Interaction Model

We study the bulk four-fermion interaction model defined by

$$\mathcal{L}^{5D} = \sqrt{-G} [\bar{\psi}_1^{5D} i \not{\partial} \psi_1^{5D} + \bar{\psi}_2^{5D} i \not{\partial} \psi_2^{5D} + \lambda (\bar{\psi}_1^{5D} \psi_2^{5D}) (\bar{\psi}_2^{5D} \psi_1^{5D})], \quad (6)$$

where we assume the existence of two kinds of bulk fermions with different parity,

$$\begin{cases} \psi_1^{5D}(x, \theta) = \gamma_5 \psi_1^{5D}(x, -\theta), \\ \psi_2^{5D}(x, \theta) = -\gamma_5 \psi_2^{5D}(x, -\theta). \end{cases} \quad (7)$$

Two kinds of fermion necessary for constructing Dirac mass term on the brane. It is possible to consider the other types of four-fermion interaction, for example  $(\bar{\psi}_1^{5D} \psi_1^{5D})(\bar{\psi}_1^{5D} \psi_1^{5D})$  and  $(\bar{\psi}_2^{5D} \psi_2^{5D})(\bar{\psi}_2^{5D} \psi_2^{5D})$ . But the interaction in Eq.(6) is essential to generate a low mass mode.

Following the procedure in Ref. 8 we derive the mode expansion of the bulk fermion in the RS background.

$$\psi^{5D}(x, \theta) = \sum_{n=0}^{\infty} \psi_R^{(n)}(x) g_R^{(n)}(\theta) + \psi_L^{(n)}(x) g_L^{(n)}(\theta), \quad (8)$$

where  $g_L^{(n)}$  and  $g_R^{(n)}$  are left and right mode functions which satisfy

$$\begin{cases} \int d\theta e^{-3kr|\theta|} g_L^{(n)}(\theta) g_L^{(m)}(\theta) = \delta_{mn}, \\ \int d\theta e^{-3kr|\theta|} g_R^{(n)}(\theta) g_R^{(m)}(\theta) = \delta_{mn}. \end{cases} \quad (9)$$

In practical calculation it is more convenient to introduce auxiliary field  $\sigma \sim \bar{\psi}_1 \psi_2$ . Applying the KK mode expansions (8), the Lagrangian (6) reads

$$\begin{aligned} \mathcal{L} = & \bar{\psi}_{1R}^{(0)} i \gamma^\mu \partial_\mu \psi_{1R}^{(0)} + \bar{\psi}_{2L}^{(0)} i \gamma^\mu \partial_\mu \psi_{2L}^{(0)} \\ & + \sum_{1 \leq n} \left[ \bar{\psi}_{1R}^{(n)} i \gamma^\mu \partial_\mu \psi_{1R}^{(n)} + \bar{\psi}_{1L}^{(n)} i \gamma^\mu \partial_\mu \psi_{1L}^{(n)} + \bar{\psi}_{2R}^{(n)} i \gamma^\mu \partial_\mu \psi_{2R}^{(n)} + \bar{\psi}_{2L}^{(n)} i \gamma^\mu \partial_\mu \psi_{2L}^{(n)} \right] \end{aligned}$$

$$+ \sum_{m,n=0}^{\infty} \begin{pmatrix} \bar{\psi}_{1R}^{(m)} & \bar{\psi}_{2R}^{(m)} & \bar{\psi}_{1L}^{(m)} & \bar{\psi}_{2L}^{(m)} \end{pmatrix} M \begin{pmatrix} \psi_{1R}^{(n)} \\ \psi_{2R}^{(n)} \\ \psi_{1L}^{(n)} \\ \psi_{2L}^{(n)} \end{pmatrix} - \int d\theta r \sqrt{-G} \frac{|\sigma|^2}{\lambda}. \quad (10)$$

$M$  corresponds to the fermion mass. It is a function of the vacuum expectation value of  $\sigma$  and the mode functions.<sup>7</sup>

### 3. Dynamically Generated Fermion Mass

To obtain the fermion mass in four dimensions we need to calculate the vacuum expectation value of  $\sigma$  which is determined by observing the minimum of the induced four-dimensional effective potential. Integrating over the extra direction in Eq. (10), we obtain the induced four-dimensional theory. Since the RS background has no translational invariance along to the extra direction it is impossible to generally perform the integration over the extra direction. Here we restrict ourselves in some specific forms of the vacuum expectation value,  $\langle \sigma \rangle = v e^{kr\theta}$  and  $\langle \sigma \rangle = v$  where  $v$  is a constant parameter.

After some numerical calculations we obtain the behaviors of the effective potential in both the cases and find the critical coupling where the vacuum expectation value disappears.<sup>7</sup> In Fig. 3 we draw the behavior of the critical coupling.  $\hat{\lambda}$  is defined by  $\hat{\lambda} \equiv (1 - e^{-4kr\pi})\lambda/(4k)$  and  $N_{kk}$  is a trun-

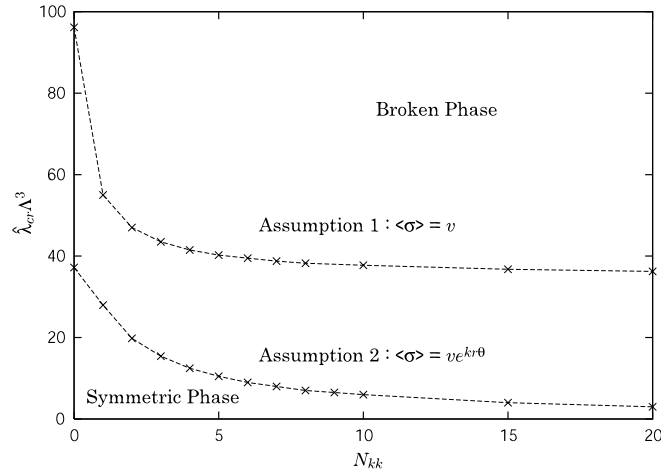


Figure 3. Critical coupling constant as a function of the truncated scale  $N_{kk}$ .

cated scale of KK mode summations. It is natural to take  $N_{kk} \sim O(10^{16})$ . In the region between two critical lines the state,  $\langle \sigma \rangle = ve^{kr\theta}$ , is more stable than  $\theta$ -independent one. For a large  $N_{kk}$  limit the critical coupling is proportional to  $1/N_{kk}$  in the  $\langle \sigma \rangle = ve^{kr\theta}$  case. But the critical coupling seems to be a constant,  $\hat{\lambda}_{cr} \sim O(30)/\Lambda^3$ , in the  $\theta$ -independent case. We conclude that the natural scale of the four-fermion coupling,  $\hat{\lambda} \sim 1/M_{pl}^3$ , is located between two critical lines and  $\theta$ -dependent vacuum is realized.

For  $\langle \sigma \rangle = ve^{kr\theta}$  the fermion mass matrix of the induced four-dimensional theory reduces to

$$M = \begin{pmatrix} m_n & -v \\ -v & m_n \end{pmatrix}, \quad (11)$$

where  $m_n$  is given by

$$m_n = \frac{nk\pi}{e^{k\pi r} - 1}, \quad n = \dots, -2, -1, 0, 1, 2, \dots \quad (12)$$

The eigen values of the mass matrix is described as

$$m_f \sim \left| v + \frac{nk\pi}{e^{k\pi r} - 1} \right|, \quad n = \dots, -2, -1, 0, 1, 2, \dots \quad (13)$$

It corresponds to the mass for each KK modes in four dimensions. We can choose  $n$  where  $m_f$  is smaller than  $k\pi/(e^{k\pi r} - 1) \sim M_{EW}$ . There is a mode whose mass is smaller than the electroweak scale,  $M_{EW}$ , even if  $v$  develops a value near the Planck scale. Therefore a low mass fermion is generated dynamically in the bulk four-fermion interaction model.

#### 4. Conclusion

The dynamical origin of the electroweak mass scale have been investigated in the RS background. We have assumed the existence of two kinds of bulk fermion fields with different parity and studied a bulk four-fermion interaction model. Evaluating the effective potential for two specific  $\theta$ -dependence of the state, we have calculated the critical value of the four-fermion coupling and found the more stable state. In a natural choice of all the physical parameters the vacuum expectation value depends on the extra direction. In the stable state the fermion mass term has been analytically calculated. We have shown the existence of a mode whose mass is smaller than the electroweak scale. The electroweak mass scale can be realized from only the Planck scale in the RS brane world due to the fermion and anti-fermion condensation. This is one of the dynamical realizations of the so-called Randall-Sundrum mechanism.

There are some remaining problems. We consider only two specific forms of the vacuum state and conclude the state whose expectation value of  $\sigma$  has the form  $ve^{kr\theta}$  is more stable. To find the true vacuum we must calculate the induced effective potential for a general form of  $\langle\sigma\rangle$ .

The fermion and anti-fermion condensation may affect the structure of spacetime. To analyze the spacetime evolution the behavior of the stress tensor is under investigation.

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